In recent years Teacher Fellow schemes have been run both by the Institute of Physics and the Royal Society of Chemistry in Britain; however to date a similar national scheme has not been available in Wales for mathematics. It was therefore decided by the National HE STEM Programme in Wales to run a project whereby sixth form teachers of mathematics would be given the opportunity of spending spring term 2012 (either full or part time) in a Welsh university mathematics department. The aim of this project was to explore issues surrounding student transition between school/college and first year university. There would also be an exchange of knowledge between the school teachers and the university lecturers – the teacher would be able to take back to their school the university view, and the university would have the benefit of gaining knowledge from a current classroom teacher.

These secondments were advertised across Wales and the border regions to both sixth form schools and FE colleges. A number of teachers were interviewed and two Teacher Fellows were appointed:

Mr Peredur Powell – Head of Mathematics at Ysgol Gyfun Bro Morgannwg accepted a full time secondment at the School of Mathematics, Cardiff University.

Mr Peter Burford – Mathematics Lecturer at Hartpury College accepted a part time secondment at the Division of Mathematics and Statistics, University of Glamorgan.

Interlinked to this project was ‘Shadowing to Promote Dialogue’ whereby postgraduate students from Cardiff University and undergraduates and postgraduates from Aberystwyth University visited secondary schools and FE colleges to discuss their own experiences of studying mathematics at HE level and through the use of questionnaires gathered information on student/teacher expectations of studying university mathematics. Focus groups were also held in Aberystwyth University to explore transition issues experienced by undergraduates from different level three backgrounds.
Cardiff School of Mathematics offers research-led teaching on the following undergraduate degree programmes:

- MMath Mathematics
- BSc Mathematics
- BSc Mathematics and Its Applications
- BSc Mathematics, Operational Research and Statistics

as well as a number of Joint Honours degree programmes which combine the study of mathematics with another discipline. Students on the BSc Mathematics and its Applications, and the BSc Mathematics, Operational Research and Statistics courses also have the opportunity to spend a ‘year in industry’ as a salaried employee. The School has an intake of approximately 160 students each year predominantly from an A level background. Typical admissions requirements are AAB (or equivalent) to include a grade A in either Maths or Pure Maths. In recent years the proportion of students who have studied Further Maths at A level has increased (about one third of the 2011-12 intake achieved a grade A* - C in A level Further Maths). Outreach activities organised by the School include running general ‘Why study maths?’ sessions through to specific topics such as game theory and mathematical modelling for epidemiology. Despite the strong academic background of the student intake, it is apparent that some students are not sufficiently prepared for the challenge and rigour of studying mathematics at university. A great deal of support is already in place to try and ease the transition to university study such as School induction events, refresher classes, and a university-wide maths support service.

The Division of Mathematics and Statistics at Glamorgan University offers research-led teaching on the following undergraduate degree programmes:

- BSc Mathematics
- BSc Mathematics and Its Applications
- BSc Computing Mathematics
- BSc Financial Mathematics

It also supports the maths components of a wide range of major minor and joint degrees as well as the combined study options.

In addition to providing the above course choices, the Division’s undergraduates have the option of doing a sandwich year out in industry or commerce. The first year intake is currently between 70 and 80 students and, for the main maths course, which consists of the vast majority of students, BBB grades are required with at least a B in Maths. Pleasingly there are a significant number of students in the first year with a grade A in Maths. As regards other smaller courses, Glamorgan will consider those students with a grade C in Maths but these are limited to very small numbers and the aim is to support mature students or those from difficult backgrounds.

To help all maths students make a successful transition to studying for a degree, a suite of seven outreach activities from KS3 to KS5 runs at Glamorgan. These provide students with some support throughout their time in school and enable them to become thoroughly familiar with what a university is like. Weekly tutorials are run for small groups of students in the first year.
How can schools and colleges help to prepare their students

Below are a number of suggestions which have arisen as a result of the teacher fellows working in the university departments and the Shadowing to Promote Dialogue project described in the Introduction.

University Open Days
Heads of department need to keep abreast of university open days and strongly encourage their students to attend. They could also help their students by being familiar with admission criteria and by highlighting the various undergraduate mathematics courses. Open days are now much better focused and clearly thought out than they were and virtually every sixth former is accompanied by a parent (or two) who can also ask questions. With competition so fierce for university places it benefits the forthcoming undergraduate to know when the open days are and who to contact.

The Shadowing to Promote Dialogue Project found that ‘University open days successfully help to prepare students for what to expect at university. In particular they are useful in providing more tailored, personal advice. Students should be encouraged to attend open days whenever possible, providing their current education is not negatively impacted upon’. (Key Report Finding).

Links with a local university
Schools and universities need to communicate clearly and establish an effective relationship so that opportunities can be brokered for members of the sixth form to go to talks and lectures at the university and for university students or lecturers to visit local schools. This could also facilitate the sharing of ideas and information between teachers and lecturers concerning the respective curricula.

Amount of hard work
School teachers should impress upon their students how much hard work is required to succeed at university level. Students typically study 120 credits across an academic year and one credit is equivalent to at least 10 hours of work (lecture and study time), therefore a minimum of 1200 hours of work is required. This is the equivalent of working Monday to Friday, eight hours a day for 30 weeks.

Independent learning
Often, the default position at A level is for teachers to bend over backwards to assist their pupils including chasing up any work lost. For instance teachers may give unacknowledged extra lessons during the dinner hour or after school if some pupils have been absent or provide extra examples or notes tailored to an individual. Such assistance is expected by the student, their parents and the school. Whilst a university has very good and effective methods of assisting and supporting its students, the personal responsibility for completing work satisfactorily and on time is deemed to fall on the shoulders of the students. The default position here is for the student to help his or herself by making the most of the support that is available. Teachers need to encourage pupils to take responsibility for their own work during their courses.

These points were backed up by the Shadowing Project (Cardiff) which noted that ‘FE students are generally not taught to be independent thinkers. This is largely due to the strong result-driven culture faced by teachers who find it difficult to strike a balance between examination results and helping students to prepare for university. Universities should be made aware of the pressure
that teachers are under to achieve good quality grades from their students and that helping students to prepare for university suffers as a result’. (Key Report Finding). However, in Aberystwyth, students thought that FE colleges encouraged more learner autonomy and the development of independent learning skills.

Open questions
A level papers with their current lack of variety from one paper to the next year on year, together with multiple re-sit opportunities, can stifle conceptual understanding. Until the curricula are modified, the teacher could help to remedy this by providing a context where more open questions can be discussed. This would help the most able (who typically study mathematics or a related discipline at university level) on their transition to higher education.

Studying A Level Further Mathematics
The study of further mathematics should be encouraged. If it is not possible to do so in a particular school then teaching of further mathematics should be encouraged by video-link or distance learning. There is evidence to suggest that those students who study further mathematics find the transition to university level work smoother than those who have studied mathematics or pure mathematics A level.

Self study modules to prepare for university mathematics
Students should be encouraged to prepare for study at university by following a suitable self study module during the summer preceding university entry.

A picture of university life
It would help students as they prepare for their first year at university to be told that:
• Up to five or six mathematics lecturers are likely to be teaching them possibly in teams for a single module.
• Much of the module content will have been written by the lecturers themselves who might subsequently make the learning materials available on the virtual learning environment (VLE) of the university.
• Students are expected to undertake additional reading of material outside of formal lectures. This might include VLE content, textbooks and other materials.
• Each lecturer will create the assessment criteria for their own modules.
• Feedback on their work will often take longer than students might have been accustomed to during their A level studies. Support may well not be immediate.
• Students are encouraged to approach lecturers to discuss ideas; however private tuition is not common.
• Resits are often held in the summer holidays.
• Lecturers spend less time marking individual work; often outline solutions are provided to encourage self-reflection.

The Shadowing Project backed up the need for students to receive further information on university life:- ‘Despite a general overall good grasp of what to expect at HE, it was observed that students are actually less knowledgeable and less aware than they perceive themselves to be. This was particularly pertinent with respect to assessment types and penalties imposed for late submissions’. (Key Report Finding).
What can university maths departments do to help students overcome transition issues?

Diagnostic testing
The initial diagnostic test is a good method for probing the students’ strengths and weaknesses, in term of procedural mathematics and algebraic fluency. There may also be room for another test that probes for deeper understanding at a conceptual level.

Lecture room technology
Most students are now comfortable with newer technologies which are used extensively in schools and colleges so it is appropriate to use the resources in the lecture theatres and teaching rooms e.g. visual aids via hyperlinks. In those rooms where technology is lacking make a case within the department for upgrading it.

Teaching techniques
There sometimes seems to be an over reliance on lecture notes as the prime way of teaching. This is understandable of course given the sheer number of students present in some lectures. Other methods such as visual links (see above) could also be encouraged and, difficult though it may be, room for at least some questioning and discussion should be found. Variation of method even if introduced fleetingly every 15 minutes or so will improve student engagement and understanding.

Tutorial sessions however work well, when individual students lead others in giving their explanations.

Basic teaching techniques are sometimes forgotten by some lecturers. Fundamental points need to be highlighted and “hammered home”. Students should be reminded of what was taught and learnt in the lecture at the end of that lecture. They should also be given “thinking time” before copying the next piece of mathematics or attempting to answer a question. Lecturers should never remain complacent over their teaching – one way of doing this is to encourage peer observation.

Technological developments
Given the students’ reliance on Android and Apple mobile phones, maths questions and proofs answered via capture software should be developed over the next two years. Recent research suggests that students respond well to this method of instruction.

Support
Study support systems in universities, particularly maths support services, are highly effective in assisting student transition. Students who experience difficulties, should be encouraged to access these services as students are often reluctant to seek help directly from teaching staff. For those departments which have an ‘open door policy’, records need to be kept of who attends and why. This data should highlight common difficulties and pick up those students who seldom attend support sessions. Students should also be reminded that it is their personal responsibility to improve their study skills.
There is a concern, often expressed in universities that the time taken before students receive feedback on their work is unduly long. If this could be given in a more timely fashion this could potentially lead to less students seeking support.

The issue of support was raised by the Shadowing Project which noted that while students were provided with considerable support at level three, they needed to make a more conscious effort to access this available support at university. If they failed to access support there was no ‘safety net’.

**A level curricula**

Lecturers need to become familiar with the current A level curricula for mathematics. Sometimes topics are taken for granted as having been taught at A level even though they are no longer in the syllabus. This leads to frustration on the part of the lecturer and confusion on the part of the student. Consistency in symbolism should also be introduced at an early stage.

**Induction courses**

The possibility of an induction course should be considered in order to address fundamental notions of proof, logic, mathematical writing etc.

**Links with schools and colleges**

Departments need to be encouraged to undertake outreach and widening participation activities with local schools and colleges – these activities can be carried out on the university campus or in the school/college. Working or advisory groups to include A level mathematics teachers, university staff and local authority advisory staff should also be set up.

Activities with school students need to include generic information on university life. This point was raised as a key finding by the Shadowing Project: ‘There needs to be an emphasis on other responsibilities and life changes faced when going to university, not just the educational changes. In terms of the educational changes, it is recommended that students need a greater awareness of the level of ‘competition’ at university, as well as the possibility of failure and how to turn this into a positive’. (Key Report Finding).

**Transition policy**

A sizeable proportion of university students are in their transition year. Universities’ policy on transition to higher education cannot be static and transition issues are often immediate. A department needs to meet regularly to ‘keep its eye on the ball’.
Mr Peredur Powell – Head of Mathematics, Ysgol Gyfun Bro Morgannwg, Barry

http://www.bromorgannwg.org.uk/

‘Earlier this year I spent some 12 weeks as a STEM teaching fellow in the School of Mathematics at Cardiff University looking into the question of mathematical transition between Year 13 A level studies and the first year of undergraduate study. I was encouraged to sit in on as many lectures as I wanted, to discuss general opinions from staff and students and to take part in the tutorial teaching, learning and assessment of first year students.

My focus was on the methods of teaching and learning employed at both school and university levels and to see what pedagogical approaches could be mutually beneficial to both.

The first thing to strike me quite forcibly was the sheer size of some of the first year classes. With up to 160 students filling up the lecture hall, waiting with various degrees of enthusiasm for the next piece of digestible mathematics, one is forced to reassess any cosy notion of group teaching and open discussion like something from a 1970s campus novel by Bradbury or Lodge.

Another thing was quite clear; these students were expected to work. They might not have known it at the time and quite possibly one or two have still to wake up to that realisation, but with each 10 credit points requiring a minimum of 100 hours work, a 120 point double semester requires at least 1200 hours of lectures and study time. Returning to school in April this would be the first thing I’d impress upon my students.

A common issue that was brought to my attention by the lecturers was the impression that many first year undergraduates just “didn’t know their stuff”. The skills of basic algebraic or trigonometric fluency seemed missing. Another issue (and one of which we as schoolteachers are aware) was that, in general, the students were reluctant to “think outside of the box” or to really conceptualize the problems that they were facing. What the students preferred it seemed was a diet of spoon fed methods – an algorithm for every occasion.

In one way it’s hard to blame the students for this algorithmic preference, since it seems that the modular ‘resitable’ and predictable A level papers can be “taught to” effectively by precisely these methods.

The more I considered the matter the sharper the distinctions between procedural learning (knowing how) and conceptual learning (knowing why) became. Both are required I believe, the former to address the problem of fluency and the latter to cement understanding. Usually both are required during the same lesson, lecture or tutorial sessions.'
Dr Robert Wilson – Director of Learning and Teaching, School of Mathematics, Cardiff University

http://www.cardiff.ac.uk/maths/

‘The Teacher Fellow scheme has been an extremely positive experience from both a School perspective and a personal one. Having a fresh perspective on the pedagogical aspects of university provision from an individual who is committed to developing the student learning experience and has a detailed knowledge of the student experience prior to university has been invaluable. The recommendations that have been presented may not be totally novel, but they highlight many important and practical aspects of teaching that can quite easily be adopted to complement the approaches used to deliver the rigour of university mathematics.

Within the School, plans are already being considered on how some of the suggestions might be developed and embedded into current provision. Furthermore, the project has acted as a reminder of the importance of developing strong links with those involved in the teaching of mathematics (not just the students) prior to university study. Such a forum has existed in the past where staff from the School of Mathematics and a number of local schools/colleges were able to discuss and debate various matters. It would be a positive outcome if this project could act as a springboard for such a forum to be resurrected.’

Mr Peter Burford - Mathematics Lecturer, Hartpury College, Gloucester

http://www.hartpury.ac.uk/

‘It’s Spring now and if you will allow me to think of myself as a non-teaching temporary, (almost fleeting) associate member of the Maths/Stats department then you will understand that my primary function for the past three months has been ‘to take a view’.

As with all educational establishments, information overload is endemic. The skill it seems to me is to listen and talk often one-to-one but also in small groups, to staff of course, not students. It’s not much use talking to students, attempting to gather their views on selected topics unless they trust you and, to some extent, need you and that does not happen in thirty odd days, or at least not with me.

It helped me to ‘sit in’ most Tuesday mornings between 10 and 11 a.m. on a series of lectures given to over fifty transition year students as they prepared for an examination on differential equations. The students were busy writing notes, attempting examples, listening to the lecture and at times asking direct questions. Answers were quickly given and further explanations offered; the students were attentive, cooperative and appeared thoroughly to enjoy their time in the lecture room. Even when late, they minimized the nuisance and sat down quickly without fuss. If this group of students are a typical example of students at Glamorgan or at least students of Mathematics at Glamorgan then all should be well over the next few years. A big thank you to the lecturers and students who, over several weeks, created a most worthwhile teaching and learning experience. Now students recognize me, some smile when we pass each other, even tease me when Arsenal lose a match – they offer their opinions, hard information has started to flood in.'
Of course much more evidence is needed to find a balanced view. Should questionnaires have been distributed? – of course not. They are often in my view a considerable waste of time and possibly a slightly deceitful way to obtain trivial information. If you want hard information, read as much as possible and talk to members of staff.

Eleven full time and very busy members of staff have spent at least one hour, often two and sometimes three hours answering my questions in one-to-one meetings. With considerable patience and good humour they have allowed me to gain some insight into their working lives. These are real people with real passion, highly qualified yes but no hint of the ‘crusty academic’.

They want their students to understand and hopefully retain the Mathematics they so enjoy teaching (well mainly). My interest has been to trace if possible to what extent this is successfully and completely taking place in the transition year at Glamorgan University. My suggestions for departmental development reflect over forty years in education but so little time here. It is so easy to offend and that really is not my purpose. It would be great, for goodness sake, to be invited back sometime.'

Dr Mark Jones – Senior Lecturer and Outreach Coordinator, Division of Mathematics and Statistics, University of Glamorgan

http://fat.glam.ac.uk/subjects/maths/

'I am delighted to say that Peter’s placement has been a real success. Peter has an engaging personality and an enthusiasm for teaching maths that has enabled all of us here, both staff and students, to enjoy his company and thoroughly enjoy his time with us. As a very experienced teacher, Peter has had the wherewithal to ‘tread lightly’ and has chosen the right times to speak to lecturers and students and sit in on numerous lectures. So it is with a great sense of anticipation and enthusiasm that I look forward to reflecting on some of Peter’s considered thoughts concerning transition.

Helping students’ transition between school and university has become a bit of a ‘hot potato’ so to speak and many students seem to continue to find transition a challenge. This is true for our first year intake this year. On the Maths Degree, we currently run weekly tutor groups where small groups of first years are timetabled to meet with an academic member of staff. We also monitor attendance scrupulously in order that poor attendance levels can be nipped in the bud as it were and a meeting with the year tutor and/or scheme leader usually ensues.'
1) One such course is run by MEI (Mathematics for Education and Industry) and consists of computer aided instruction to be done by the student. MEI’s website is given below:

http://www.mei.org.uk/index.php

2) For an example of a typical first semester calculus course mapped to internet resources see Appendix B.

3) Click the following link for some downloadable exemplar questions from the Open University.

http://www.maths-screencasts.org.uk/

4) Research suggesting that screencasts can be educationally instructive can be found in:


http://mathstore.ac.uk/headocs/Connections_12_1_Jordan.pdf

5) For a breakdown of different syllabuses together with a list of what is no longer taught in the core A level modules see Appendix C.

6) See “Mind the Gap: Mathematics and the transition from A levels to physics and engineering degrees. (The Institute of Physics – July 2011). The full report is available here:


7) See Douglas Quinney’s informative article “So just what is conceptual understanding of mathematics?” (MSOR connections Volume 8 No 3 August – October 2008.) You can download the article from here:

http://mathstore.ac.uk/headocs/8302_quinney_d_conceptualmaths.pdf

8) See Appendix A for an example of how a question can be adapted to move from one side of the formative – conceptual spectrum to another.

9) The perils of just using procedural methods for teaching ordinary differential equations and to then ask questions that require conceptual understanding is meticulously documented in:

Appendix A – Peredur Powell

In brief, procedural learning is essentially learning by algorithmic rote (which of course may sometimes be required, for example in learning trigonometric identities). It is, I suppose learning how. Conceptual learning probes for deeper understanding of a topic and links knowing how with understanding why. To draw a schism between both types of learning however is to invoke a false dichotomy, since it seems that most learning happens somewhere on a spectrum between the procedural and the conceptual.

Here is a good question from a (rather old) textbook on statistics:

“When a spinning pointer with scale 0 to 1 is spun twice, the a priori model for the difference of the outcomes (first outcome minus the second) is given by the density in figure 3.9. Determine the probability that
a) The two outcomes are within 0.1 of each other
b) The outcomes differ by more than 0.5”

Basic ideas of statistics gan BW Lindgren t 70 (Collier Macmillan Publishers 1975)

This question is obviously not purely procedural since it asks the student to apply his understanding of continuous probability distributions to a novel case. To calculate the answers a bit of quick thinking is required to work out the areas enclosed- it's not just reading off the answers uncritically from a table.

However the question could be made to stand even further to the conceptual side of the spectrum if the teacher or lecturer so desired.

Consider this

When a spinning pointer with scale 0 to 1 is spun twice, the a priori model for the difference of the outcomes (first outcome minus the second) is given by the density in figure 3.9

(Note there is no figure 3.9). It's up to you to construct it.

Or this

An even more conceptual type of question would be to give students the diagram and ask them to come up with an experiment that fits it.

Which one, (if any of the three), to be asked by the lecturer is to be decided by her/his understanding of the student’s ability and knowledge of the teaching and learning styles engaged in general by the students.
MA 1000 CALCULUS (First Semester)

Useful Web Pages

These web pages are hyperlinked to specific sites. They are here in order that you may grasp some the important underlying concepts encountered on this course.

You should look at these pages before attempting the exercise sheets.

Exercise Sheet 1

1) The idea of a limit (http://www.mathcentre.ac.uk/search/?q=limits) (This links to the “mathcentre” resource page on limits. Look at the video or read the notes on “Limits of functions”
2) Differentiating polynomials (http://www.waldomaths.com/Diff2N.jsp)
3) Differentiating exponentials 1 (http://www.waldomaths.com/Diff4NL.jsp)
4) Differentiating exponentials 2 (http://www.waldomaths.com/Diff3N.jsp)

(2,3 and 4 introduce a graphical approach in order to understand the idea of differentiation as a measure of slope, indicating in particular that

\[ y = e^x = \frac{dy}{dx} \]

Exercise Sheet 2

1) Inverse trigonometric functions
(http://www.ugrad.math.ubc.ca/coursedoc/math100/notes/zoo/invtrig.html)

Get to know these well! This link will help you understand their graphs. Make sure that you also know the domain and range of each function.

2) Hyperbolic functions
(http://www.mathcentre.ac.uk/topics/graphs/hyperbolic/)

This links to various resources on “mathcentre”. The second video has earned a reputation for being a bit of a “classic”.

3) Graphs of hyperbolic functions
(http://www.waldomaths.com/Hyper1NLW.jsp)

All kinds of hyperbolic functions are to be seen here. Try graphing two simultaneously in order to see the relationship between them.

Appendix B – Peredur Powell
Exercise Sheet 3

1a) Riemann Integrals
(http://www2.seminolestate.edu/lvosbury/CalculusI_Folder/RiemannSumDemo.htm)
A visual treatment of Riemann Integrals. especially useful if you have a graphical calculator.

1b) Riemann Integrals
(http://www.geometer.org/mathcircles/riemannint.pdf)
A good introduction to the Riemann integral.

2) Integration by parts
(http://www.mathcentre.ac.uk/students/topics/integration/by-parts/)
Plenty of resources here including a quick reference guide, some notes and a video. Check out the “Calculus Refresher Guide” which covers many areas of differentiation and integration together with the answers.

Exercise Sheet 4

1) Integration by substitution (http://www.mathcentre.ac.uk/students/topics/integration/by-substitution/)
Plenty of stuff here, including an instructional video.

2) Integration using partial fractions
(http://www.mathcentre.ac.uk/students/topics/integration/by-partial-fractions/)
The two videos cover everything you need to know about integration using partial fractions. Watch these first and then try the questions on the accompanying sheets.

Exercise Sheet 5

1) Volumes of Revolution
http://library.leeds.ac.uk/tutorials/maths-solutions/
Two well explained examples on brief videos. Click on the link under integration for “Volumes of revolution”
### Appendix C - Peredur Powell

Comparison between the core syllabi for the four boards – March 2012

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<th>TOPIC</th>
<th>AQA</th>
<th>EDEXCEL</th>
<th>OCR</th>
<th>WJEC</th>
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<td>(INCLUDING THE SECOND DERIVATIVE)</td>
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<tr>
<td>Recognise and solve equations in $x$ which are quadratic in some function of $x$</td>
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<td>APPLICATION OF DIFFERENTIATION</td>
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<td>C1(AREA IN C2)</td>
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<td>ARITHMETIC PROGRESSIONS AND SERIES</td>
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<td>C2</td>
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</table>
Topics no longer taught in core courses – March 2012

• Sum and product of the roots of a quadratic equation
• Loci
• Rates of change of connected variables
• Half angle formulae
• Factor formulae (sum of two sines etc)
• Sums of powers of the natural numbers
• Mathematical induction
• Power series
• Hyperbolic functions
• Curve tracing
• Complex numbers
• Curvature
• Matrices (with the exception of those used for vectors)
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Cardiff and Vale College, Wales

Ysgol Gyfun Aberaeron, Wales